TWO CRITERIA FOR CORRELATING AND UPDATING FINITE ELEMENT MODELS

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This paper discusses the use of frequency-domain criteria for FE model correlation and updating. The criteria, which can be local or global, are either shape or amplitude based. The former are sensitive to mode shape differences but not to relative scales, and the latter depend on the actual response amplitudes. The updating objective function is formulated in such a way that two global correlation criteria are satisfied simultaneously. A sensitivity-based updating problem is expressed as a set of linear equations where the coefficient matrix contains the partial derivatives of the correlation functions with respect to the chosen design variables. The formulation has the advantage of being able to deal with incomplete measurements and there is no critical selection of updating frequency points. The correlation criteria were used successfully to quantify the initial closeness between 4 increasingly detailed FE models of an automotive oil pan. The updating of the second most detailed model using simulated response functions from the most detailed model exhibited quick convergence properties and yielded a very good match between the target and updated models. However, the model was corrected in a curve-fitting/minimisation sense since actual modelling deficiencies were compensated by adjusting a small number of pre-selected design parameters.

1. INTRODUCTION

Several review articles on FE model updating reveal a wealth of updating algorithms but success seems to remain case dependent and applicability is bounded by the skill of the analyst in choosing a *correct* updating procedure (Natke 1988, Imregun & Visser 1991, Mottershead & Friswell 1995a). In any case, two somewhat related approaches are now accepted as state-of-the-art tools: the inverse eigensensitivity method (Zhang & Lallement 1987) and the response function method (Lin & Ewins 1990, Visser & Imregun 1991). A review of the case studies using these methods unveils a fundamental problem: a particular solution is usually non-unique and a generated solution does not necessarily represent a true physical meaning (Imregun 1995). A detailed account of the state-of-the-art in finite element model updating and several numerical techniques are presented in a recent authoritative textbook by Friswell & Mottershead (1995b).

There are several issues that need to be resolved before a reliable and universally-applicable updating method can be developed: the minimum required experimental accuracy (Ziaei-Rad & Imregun 1996), the selection of optimum measurement locations (Imamovic & Ewins 1998), the treatment of discretization errors (Mottershead et al. 1992), the numerical aspects of the inverse problem (Fregolent et al. 1996, Link 1998), error localisation versus global correction and the derivation of novel elements for updating (Ahmadian et al. 1998).

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The current work will attempt to consider one such fundamental question: the determination of minimum required initial closeness between two given models so that eventual updating, by whichever method, becomes possible? Although a large number of model correlation techniques exist, there are no concise guidelines for an objective and quantitative assessment of the amount of agreement between two models. For instance, an overlay of response functions from two different models is a very useful measure of the overall qualitative agreement but it can not provide an adequate indication of whether updating will be possible in this particular case. Similarly, the model assurance criterion (Allemang & Brown 1982) can give a numerical description of the modal correlation between two given models but it lacks the precision to indicate if updating will be possible. For instance, it is not sensitive to relatively small -but sometimes crucial- changes in the mode shape, especially for large-order systems. When building an FE model, the analyst may create a number of interim versions with increasing detail, until he/she is satisfied that there is *enough* correlation with experimental data. Although it is often the case that the more detailed model will correlate better, there are no techniques to rank these various models with some numerical precision. From the above discussion, it must be clear that one needs to develop a set of precise numerical tools for comparing two models, which can be of theoretical or experimental origin. It is further desirable to use these rules during the updating process so that the success (or otherwise) of the updating procedure can also be numerically quantified. Accordingly, the aim of this paper is threefold:

(i) to develop new criteria to quantify the closeness between two given models,

(ii) to be able to distinguish between frequency regions of good and poor correlation, and

(iii) to use the developed correlation criteria as the basis of the updating objective function and to show that the procedure is applicable to practical cases where the size of the FE model is large and the number of measurement points is relatively small.

2. DEVELOPMENT OF FREQUENCY DOMAIN CRITERIA

From the outset, it was decided to focus on frequency-domain correlation criteria since such an approach was considered to have several advantages. First, errors due to the modal analysis of measured data are automatically circumvented, a big benefit when studying with structures that have high modal density and high damping. Second, it is relatively straightforward to deal with complex modes and non-linear effects during the updating stage, provided such features can also be incorporated into the theoretical analysis. Finally, there is no need to find correlated mode pairs, a task that can be a surprisingly difficult for industrial applications.

A number of frequency-domain correlation tools already exist in the literature. For instance, the frequency response assurance criterion (FRAC) for the j-th degree of freedom can be defined as (Nefske & Sung 1996, Heylen & Avitabile 1998):

$$FRAC_j = \frac{\left|\{H_X(\omega_i)\}_j{}^H\{H_A(\omega_i)\}_j\right|^2}{(\{H_X(\omega_i)\}_j{}^H\{H_X(\omega_i)\}_j)(\{H_A(\omega_i)\}_j{}^H\{H_A(\omega_i)\}_j)} \qquad (1a)$$

$\{H_A(\omega_i)\}_j$ is a vector of predicted FRF values for the j-th degree of freedom, each element representing a different frequency and $\{H_X(\omega_i)\}_j$ is the corresponding measured FRF vector. The FRAC, which returns a real value between zero and unity to indicate zero/total correlation, is somewhat analogous to the COMAC (Lieven & Ewins 1988) since it contains information about a specific measurement point for a given excitation point. Using the same notation as before, another COMAC-type frequency-domain correlation function, frequency amplitude assurance criterion (FAAC), can be defined as:

$$FAAC_j = \frac{2\left|\{H_X(\omega_i)\}_j{}^H\{H_A(\omega\ )\}_j\right|}{(\{H_X(\omega_i)\}_j{}^H\{H_X(\omega_i)\}_j)+(\{H_A(\omega_i)\}_j{}^H\{H_A(\omega_i)\}_j)} \tag{1b}$$

A MAC-type correlation indicator, the frequency domain assurance criterion (FDAC), was defined by Pascual et al. (1997) as:

$$FDAC(\omega_a,\omega_x) = \frac{\left|\{H_X(\omega_X)\}^H\{H_A(\omega_A)\}\right|^H\left|\{H_X(\omega_X)\}^H\{H_A(\omega_A)\}\right|}{(\{H_X(\omega_A)\}^H\{H_X(\omega_X)\})(\{H_A(\omega_X)\}^H\{H_A(\omega_A)\})} \tag{2a}$$

where $\{H_A(\omega_a)\}$ is a vector of predicted responses at frequency ω_A, and $\{H_X(\omega_x)\}$ is is a vector of measured responses at frequency ω_X, each individual element being associated with a different degree of freedom. The FDAC, which also returns a real value between zero and unity, is analogous to the MAC since it considers the correlation between two operating shape vectors which are measured/computed at a given frequency pair. A more advanced version, that can also correlate the phase between to complex vectors, can be defined as follows (Heylen & Avitabile 1998):

$$FDAC(\omega_a,\omega_x) = S\sqrt{\frac{\left|\{H_X(\omega_X)\}^H\{H_A(\omega_A)\}\right|^H\left|\{H_X(\omega_X)\}^H\{H_A(\omega_A)\}\right|}{(\{H_X(\omega_A)\}^H\{H_X(\omega_X)\})(\{H_A(\omega_X)\}^H\{H_A(\omega_A)\})}} \tag{2b}$$

where $S = sign(\mathrm{Re}(\{H_X(\omega_X)\}^H\{H_A(\omega_A)\})$. Such a quantity varies between –1 and 1, the absolute value indicating the amount of correlation and the sign indicating the relative phase between the FRFs that are being correlated. For instance, an FDAC value of –1 indicates 100% correlation with opposite phase.

Summarising, we can see that the FRAC and FAAC provide amplitude-based information in the spatial domain while the FDAC provides shape-based information in the frequency domain. In the former case, the output is DOF based and in the latter case it is frequency based. Ideally, we need criteria that can combine, over all available DOFs, the amplitude and shape information as a function of frequency. Although such data are inherently present in a typical overlay of the measured and predicted FRFs, there are no numerical means of quantifying the agreement.

Consider the two functions below, global shape criterion (GSC) and global amplitude criterion (GAC), which satisfy the requirements above (Grafe 1998):

$$\chi_s(\omega) = \frac{\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|^2}{\left(\{H_x(\omega)\}^H\{H_x(\omega\}\right)\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)} \tag{3}$$

$$\chi_a(\omega) = \frac{2\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)+\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}$$

Here, $\{H_A(\omega)\}$ and $\{H_X(\omega)\}$ are vectors of spatial responses at a series of DOFs *i*, which are predicted or measured at frequency ω for a fixed excitation position *j*. Subscripts *s* and *a* refer to shape and amplitude respectively. As will be seen later, these last two correlation functions are perhaps the most useful ones because they can quantify the overall agreement between two models as a function of frequency, thus highlighting frequency regions of good and poor correlation immediately. Once such global criteria are formulated, it is possible to obtain the corresponding local criteria by replacing the response vectors $\{H_A(\omega)\}$ and $\{H_X(\omega)\}$ by individual scalar response data.

Using (3), a local correlation function, the local amplitude criterion (LAC), can be defined as:

$$\chi_{aij}(\omega_k) = \frac{2\left|H^*_{Xij}(\omega_k)H_{Aij}(\omega_k)\right|}{\left(H^*_{Xij}(\omega_k)H_{Xij}(\omega_k)\right)+\left(H^*_{Aij}(\omega_k)H_{Aij}(\omega_k)\right)} \tag{4}$$

where i and j are the response and excitation co-ordinates, $\{H_{Aij}(\omega_k)\}$ is the predicted FRF value and $\{H_{Xij}(\omega_k)\}$ is the corresponding measured FRF value, both at frequency ω_k, * denoting complex conjugate. Clearly, the LAC can be computed at each frequency of interest in order to obtain a numerical value of the shape correlation as a function of frequency throughout the measurement range. Although a similar treatment can also be applied to the global shape criterion, the outcome is unlikely to be useful since, as can be seen from (3), the local shape criterion will have a constant value of unity throughout the frequency range.

Based on (4), the averaged LAC can be defined as:

$$\overline{\chi_{aij}} = \frac{1}{N}\sum_{k=1}^{N}\chi_{aij}(\omega_k) \tag{5}$$

Since the averaging is over the actual response values at each frequency, one can consider in turn each DOF i of interest, the excitation position j remaining fixed during the averaging process. Such a route is an alternative to assessing the DOF correlation via COMAC.

3. CASE STUDY FOR CORRELATION: AUTOMOTIVE OIL PAN

The use of the correlation criteria of **Section 2** will now be demonstrated in the case of a practical engineering structure, a mild steel automotive oil pan. As shown in **Fig. 1**, four different FE models of increasing complexity, the so-called Levels 1 to 4, were generated using 4-node quadrilateral shell elements only (**Table 1**).

Table 1. Summary of discretization levels

Level	No of elements	No of nodes	No of DOFs
1	212	188	1,128
2	705	507	3,042
3	1,448	1,042	6,252
4	2,380	1,855	11,130

The natural frequencies obtained from the 4 models, together with the relative deviation from the Level 4 model, are listed in **Table 2**.

Table 2. Natural frequencies (Hz) predicted by Level 1-4 models and relative error (%) with respect to the Level 4 model

Mode	Level 1	Rel error Re: Level 4	Level 2	Rel error Re: Level 4	Level 3	Rel error Re: Level 4	Level 4
1	261.7	7.9	275.3	13.6	249.3	2.8	242.4
2	316.0	10.3	306.3	6.9	290.5	1.4	286.6
3	462.3	15.1	454.8	13.2	417.0	3.9	401.6
4	470.1	3.4	510.9	12.4	466.6	2.6	454.5
5	663.9	16.6	612.5	7.6	581.9	2.2	569.4
6	729.7	15.6	638.8	8.3	645.1	2.2	631.2
7	792.7	9.3	807.0	11.3	744.1	2.6	725.3
8	817.2	7.6	860.2	13.3	779.6	2.7	759.0
4	968.3	20.5	904.1	12.5	822.7	2.3	803.8
Aver error	-	11.8	-	11.0	-	2.5	-

3.1 Quantification of initial closeness

To quantify the closeness of the Level 1, 2 and 3 models to the Level 4 (reference) model, 18 Z direction response functions, computed at each of the 18 common nodes between the 4 models, were generated for each model. Typical point FRF comparisons are given in **Fig. 2.** In this format, it is difficult to quantify the amount of agreement, though it is seen that there is considerable improvement with increasing modelling detail. Furthermore, the FRF comparison of Level 3 and Level 4 models indicates a good dynamic behaviour match for the 500-570 Hz frequency range.

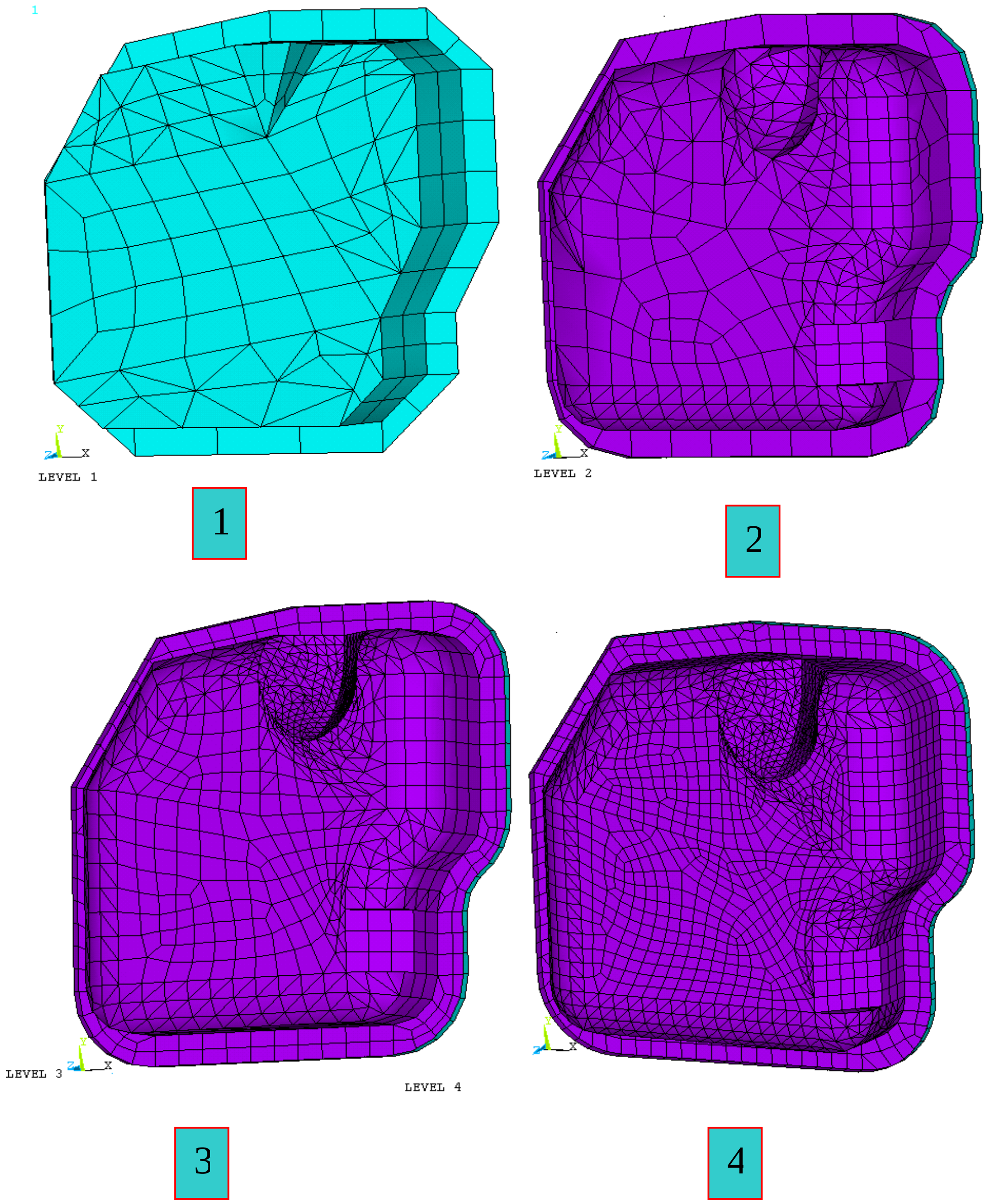


Fig. 1 FE models of four levels with increasing mesh refinement

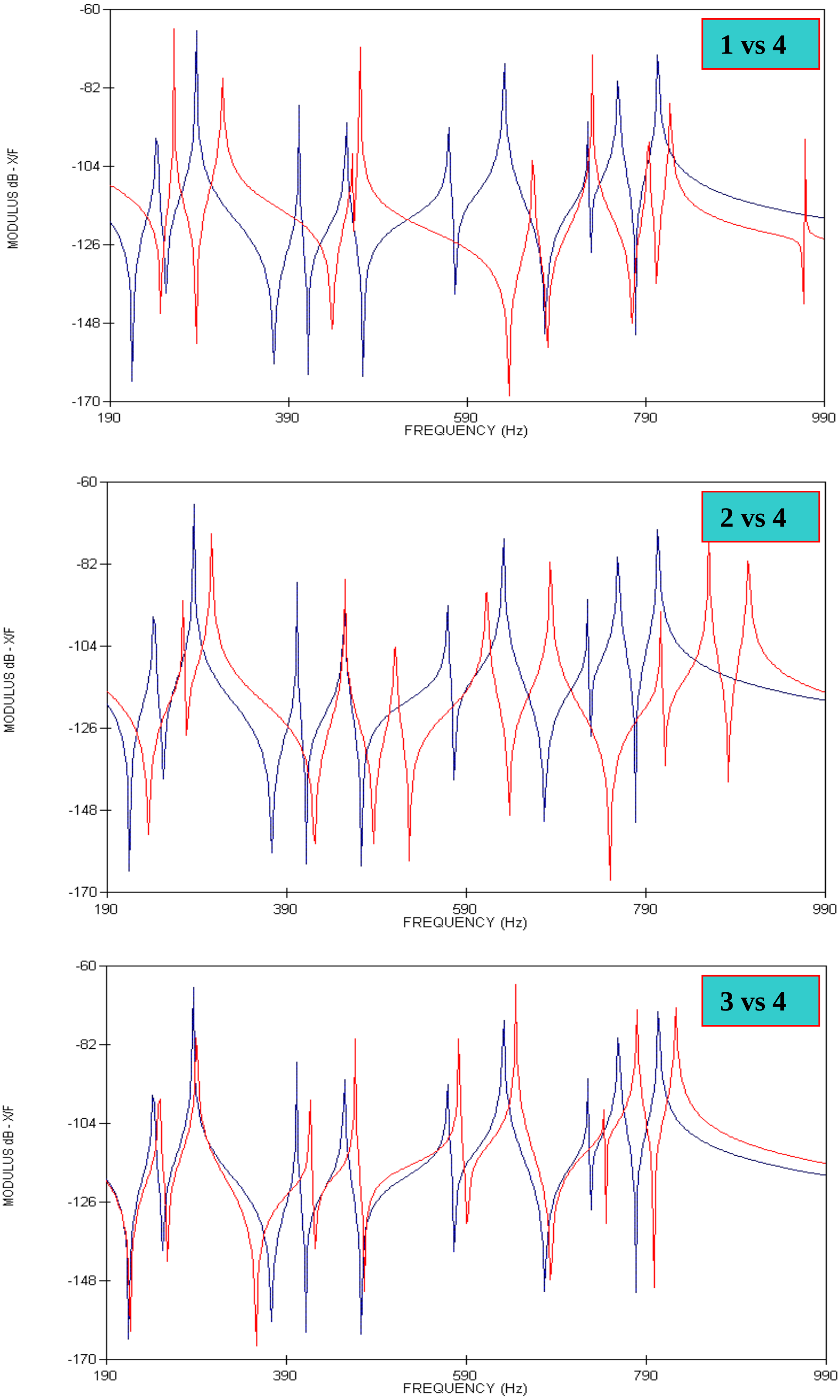


Fig. 2 Comparison of measured and initially-predicted point FRFs for various modelling levels

The corresponding point LACs are plotted in **Fig. 3** for the same three cases. The previous two features, i. e. the general improvement with modelling detail and the emergence of the 500-570 Hz frequency range as the best agreement window, can now be observed by means of an objective numerical indicator, the LAC function.

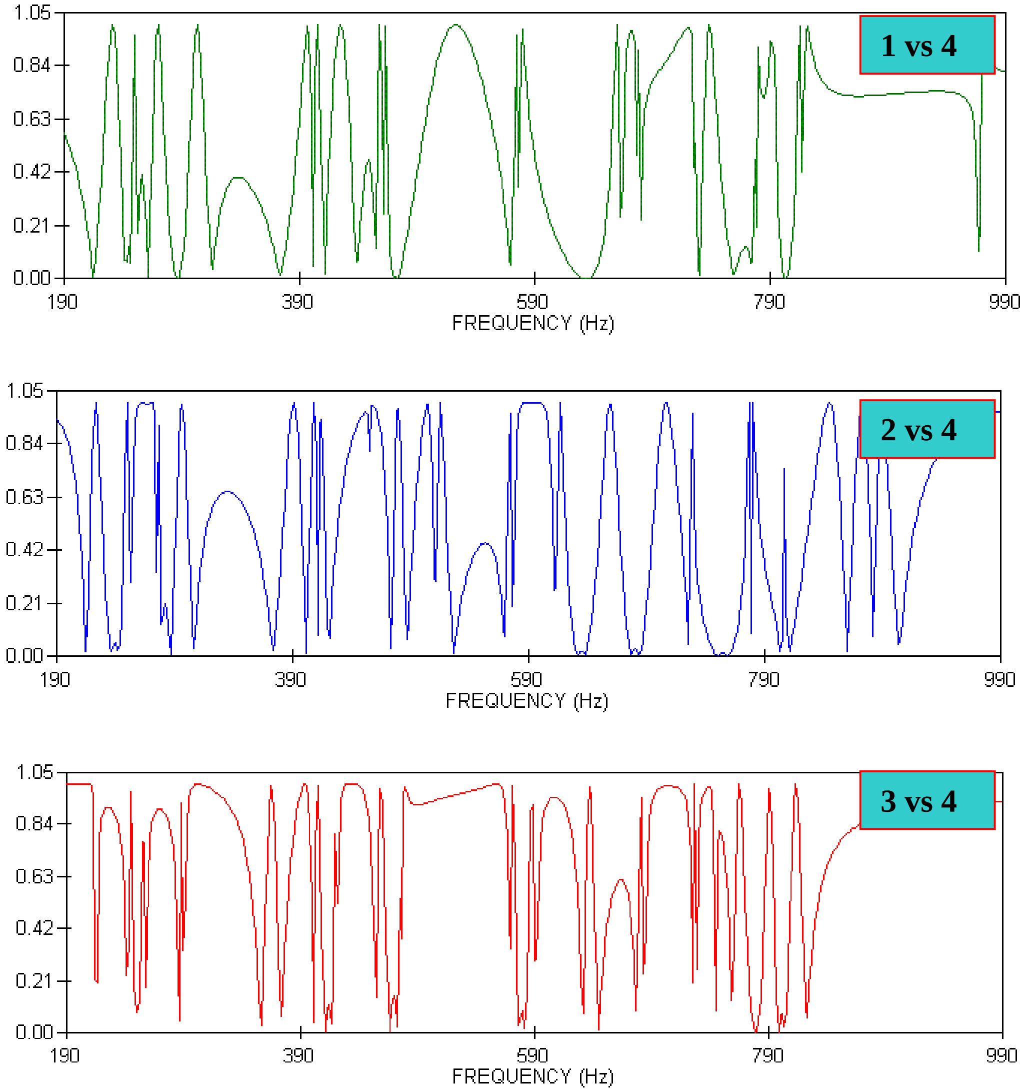


Fig. 3 Comparison of point LACs for various modelling levels

The same comparison was carried in the spatial domain by computing the FRAC, FAAC and averaged LAC criteria for each of the 18 common points between the four models. The results, shown in **Fig. 4**, indicate a similar trend, namely the numerical quantification of the improved correlation with increasing modelling detail. Also plotted in **Fig. 4** is the better known DOF correlation criterion, COMAC. For the particular structure under study, the COMAC criterion is seen to be less sensitive than the other criteria, though the poor agreement around DOF 14 is detected by all four methods.

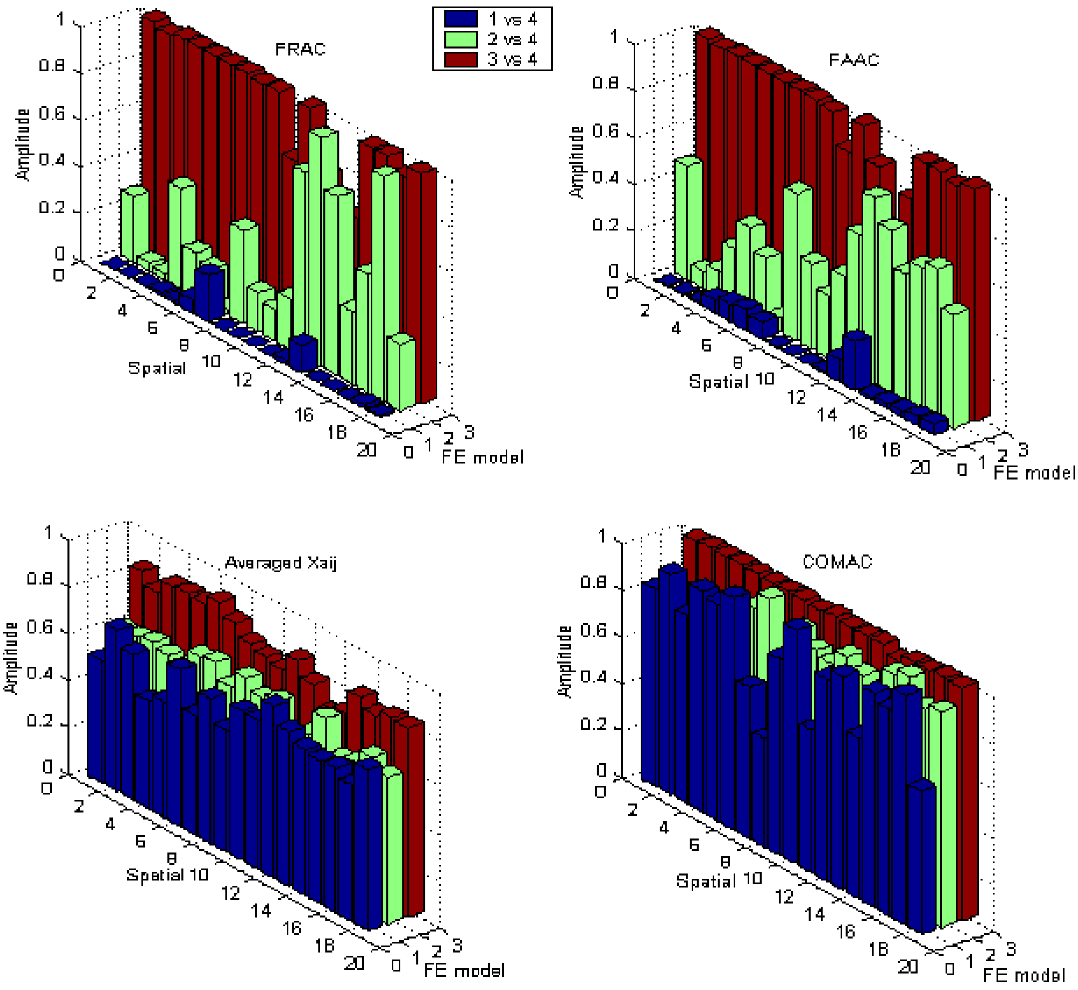
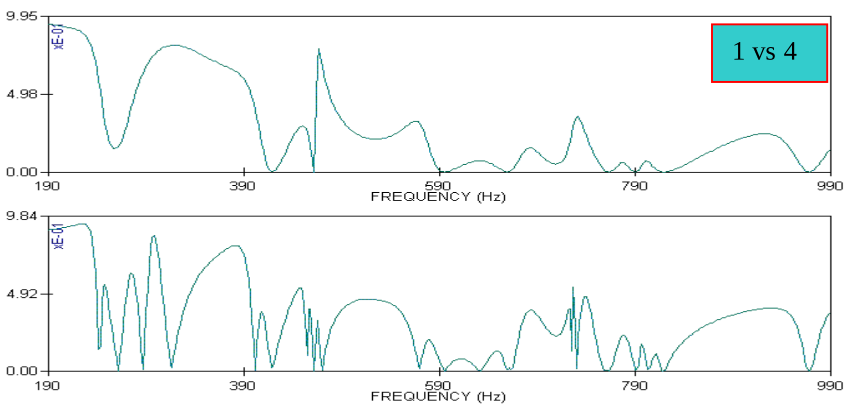


Fig. 4 FRAC, FAAC, averaged LAC and COMAC computed for the 18 common points between the four models



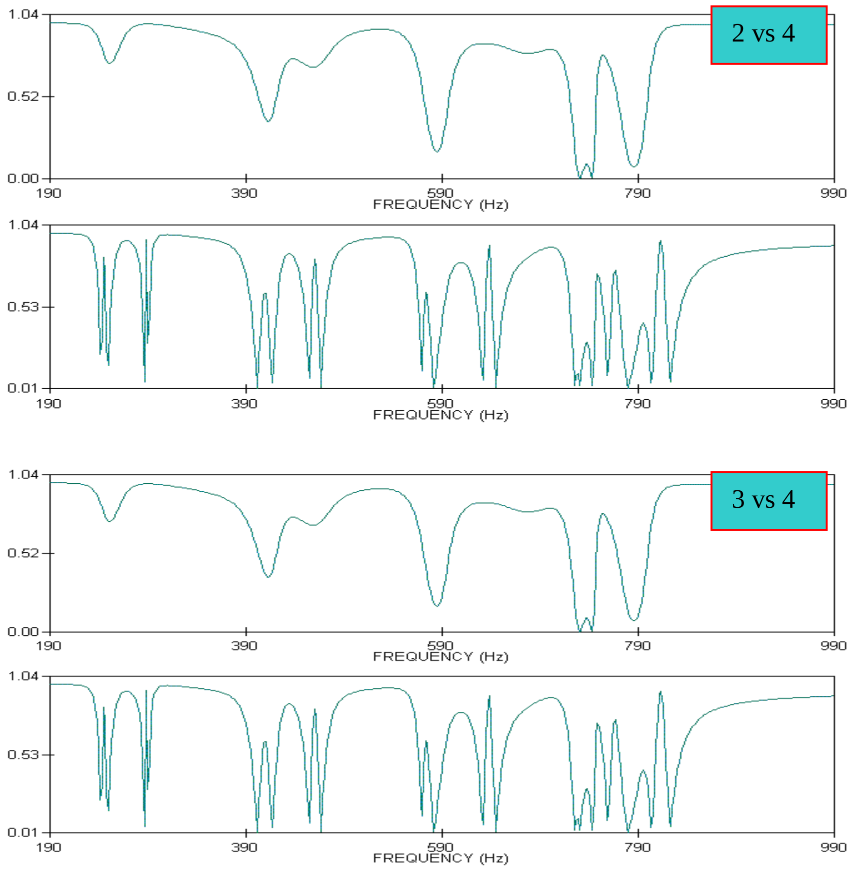


Fig. 5 Global correlation functions GSC (upper plot) and GAC for various modelling levels

To complete the assessment of the initial closeness, the two global criteria, the GSC and GAC, are plotted in **Fig. 5** for the same three cases. This particular comparison format is perhaps the most useful one since the correlation information, which is displayed as a function of frequency, contains contributions from all 18 common DOFs. As before, the overall improvement with increasing modelling detail, and the frequency ranges of good/poor agreement, can clearly be seen from the two global correlation functions.

In summary, it is seen that the frequency-domain correlation functions provide a very useful means of quantifying the initial closeness. Therefore, it can be speculated that they can also be used for model updating purposes by incorporating them into the objective functions to be minimised. After some numerical experience, it should be possible to define general rules, giving minimum criteria values below which updating should not be attempted. The fact that the same criteria are used for both correlation and updating should facilitate such a route which should also yield better convergence properties. Such ideas will be pursued in the next sections, though it is unlikely that general rules can be devised on the strength of a single case study.

4. A CORRELATION FUNCTION BASED FORMULATION FOR MODEL UPDATING

4.1 Derivation of the objective function

The correlation results of the previous section indicate that the global correlation functions, GSC and GAC, are reliable indicators of model closeness. Therefore, it is now proposed to formulate an updating algorithm that is based on these two functions. Both functions depend on the structure's geometric and material properties, the so-called design variables that will be denoted by φ. The change in χ_a due to changes in the φ parameters can be written as:

$$\Delta\chi_a(\omega_k) = \frac{\partial\chi_a(\omega_k)}{\partial\varphi_1}\Delta\varphi_1 + \frac{\partial\chi_a(\omega_k)}{\partial\varphi_2}\Delta\varphi_2 + \frac{\partial\chi_a(\omega_k)}{\partial\varphi_3}\Delta\varphi_3 + \cdots \tag{6}$$

where ω_k is the frequency at which the GAC, χ_a , is defined. The aim is to modify the design parameters φ_i in such a way that full correlation is obtained when:

$$\chi_a + \Delta\chi_a = 1 \tag{7}$$

Equation (6) becomes:

$$1 - \chi_a(\omega_k) = \frac{\partial\chi_a(\omega_k)}{\partial\varphi_1}\Delta\varphi_1 + \frac{\partial\chi_a(\omega_k)}{\partial\varphi_2}\Delta\varphi_2 + \frac{\partial\chi_a(\omega_k)}{\partial\varphi_3}\Delta\varphi_3 + \cdots \tag{8}$$

A similar expression can also be written for χ_S. Combining the equations for χ_a and χ_S, one obtains a sensitivity-based updating formulation (Grafe 1998):

$$\begin{bmatrix} \frac{\partial\chi_S(\omega)}{\partial\varphi_1} & \frac{\partial\chi_S(\omega)}{\partial\varphi_2} & \cdots & \frac{\partial\chi_S(\omega)}{\partial\varphi_p} \\ \frac{\partial\chi_a(\omega)}{\partial\varphi_1} & \frac{\partial\chi_a(\omega)}{\partial\varphi_2} & \cdots & \frac{\partial\chi_a(\omega)}{\partial\varphi_p} \end{bmatrix}_{2\times p} \begin{Bmatrix} \Delta\varphi_1 \\ \Delta\varphi_2 \\ \vdots \\ \Delta\varphi_p \end{Bmatrix} = \begin{Bmatrix} 1-\chi_S(\omega) \\ 1-\chi_a(\omega) \end{Bmatrix} \tag{9}$$

Equation (9) has a number of important features:

(i) Since the correlation functions can be formulated for any number of common DOFs, the size mismatch between the theoretical and experimental models is no longer a problem.

(ii) The size of the sensitivity matrix is the product of the number of correlation functions (here 2) and the number of design parameters, p. To be able to solve for p independent design parameters one must either define more correlation functions, or to write the existing 2 functions at N_f frequency points such that $2\ x N_f \geq p$. The latter approach will be adopted here by using several frequency points.

(iii) The simultaneous use of two such functions ensures numerical robustness. The first criterion, χ_S, ensures a global compatibility between the mode shapes while the second criterion, χ_S, refines the shape match by imposing an amplitude condition.

4.2 Use of weighting matrices

In compact form, (9) can be written as:

$$[S]\{\Delta\varphi\} = \{\varepsilon\} \tag{10}$$

It is now possible to define an objective function by using an extended weighted least-squares approach.

$$J = \{\Delta\varphi\}^T [W_\varphi]\{\Delta\varphi\} + \{\varepsilon\}^T [W_f]\{\varepsilon\} \tag{11}$$

where $[W_\varphi]$ and $[W_f]$ are the diagonal weighting matrices.

A solution of (11) is given by Link (1998):

$$\{\Delta\varphi\} = \left[[S]^T[W_f][S] + [W_\varphi]\right]^{-1}[S]^T[W_f]\{\varepsilon\} \tag{12}$$

Since the correlation functions are a measure of the closeness of the FE model and measured data, they can also be used to define the weighting matrix $[W_f]$:

$$[W_f] = \begin{bmatrix} \begin{bmatrix} \ddots & & \\ & [\chi_s(\omega)] & \\ & & \ddots \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} \ddots & & \\ & [\chi_a(\omega)] & \\ & & \ddots \end{bmatrix} \end{bmatrix} \tag{13}$$

Such a definition will allow to allocate a higher weighting to those frequencies at which the two models are better correlated. Perhaps more importantly, frequencies at which there is little correlation are effectively removed because of the low χ_a and χ_s values. Following the approach by Link (1998), the second weight matrix $[W_\varphi]$ can be defined as:

$$[W_\varphi] = \frac{\|[w_\varphi]\|_2}{\max(diag([w_\varphi]))} \begin{bmatrix} \ddots & & \\ & diag([w_\varphi]) & \\ & & \ddots \end{bmatrix} \tag{14}$$

where

$$[w_\varphi] = \left[[S]^T[W_f][S]\right]^{-1}$$

3.2 Calculation of correlation coefficient sensitivities

We now need to calculate the partial derivatives of the two correlation functions with respect to the design parameters. Using (3) one obtains:

$$\frac{\partial\chi_s(\omega)}{\partial\varphi}=\frac{\partial\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|^2}{\partial\varphi}\frac{\{H_x(\omega)\}^H\{H_x(\omega)\}\{H_A(\omega)\}^H\{H_A(\omega)\}}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)^2\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)^2}$$
$$-\frac{\partial\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}{\partial\varphi}\frac{\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|^2}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)^2\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)^2} \tag{15a}$$

$$\frac{\partial\chi_a(\omega)}{\partial\varphi}=2\frac{\partial\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|}{\partial\varphi}\frac{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)+\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)+\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}$$
$$-2\frac{\partial\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}{\partial\varphi}\frac{\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)+\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)} \tag{15b}$$

Considering the real and imaginary parts of the response separately, equations (15) can be written as:

(16)

$$\frac{\partial\chi_a}{\partial\varphi}=\frac{2}{\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|\left(\{H_x(\omega)\}^H\{H_x(\omega)\}+\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}\bullet$$
$$\left[\begin{array}{l}\mathrm{Re}\left(\{H_x(\omega)\}^H\{H_A(\omega)\}\right)\left(\mathrm{Re}(\{H_x(\omega)\}^H)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}-\mathrm{Im}\left(\{H_x(\omega)\}^H\right)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\\ +\mathrm{Im}\left(\{H_x(\omega)\}^H\{H_A(\omega)\}\right)\left(\mathrm{Re}(\{H_x(\omega)\}^H)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}+\mathrm{Im}\left(\{H_x(\omega)\}^H\right)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\\ +\frac{2\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|^2}{\{H_x(\omega)\}^H\{H_x(\omega)\}+\{H_A(\omega)\}^H\{H_A(\omega)\}}\left(\mathrm{Im}\left(\{H_A(\omega)\}^H\right)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}-\mathrm{Re}\left(\{H_A(\omega)\}^H\right)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\end{array}\right]$$

$$\frac{\partial\chi_s}{\partial\varphi}=\frac{2}{\left(\{H_x(\omega)\}^H\{H_x(\omega)\}\right)\left(\{H_A(\omega)\}^H\{H_A(\omega)\}\right)}\bullet$$
$$\left[\begin{array}{l}\mathrm{Re}\left(\{H_x(\omega)\}^H\{H_A(\omega)\}\right)\left(\mathrm{Re}(\{H_x(\omega)\}^H)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}-\mathrm{Im}\left(\{H_x(\omega)\}^H\right)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\\ +\mathrm{Im}\left(\{H_x(\omega)\}^H\{H_A(\omega)\}\right)\left(\mathrm{Re}(\{H_x(\omega)\}^H)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}+\mathrm{Im}\left(\{H_x(\omega)\}^H\right)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\\ +\frac{\left|\{H_x(\omega)\}^H\{H_A(\omega)\}\right|^2}{\{H_A(\omega)\}^H\{H_A(\omega)\}}\left(\mathrm{Im}\left(\{H_A(\omega)\}^H\right)\frac{\partial\mathrm{Im}\left(\{H_A(\omega)\}\right)}{\partial\varphi}-\mathrm{Re}\left(\{H_A(\omega)\}^H\right)\frac{\partial\mathrm{Re}\left(\{H_A(\omega)\}\right)}{\partial\varphi}\right)\end{array}\right]$$

The partial derivatives of the global correlation functions have now been expressed in terms of the partial derivatives of predicted frequency response functions. Two methods are available for their evaluation:

Method 1.

Starting from the identity $[H_A(\omega)][Z_A(\omega)]=[I]$, one can show that:

$$\frac{\partial[H_A(\omega)]}{\partial\varphi}=-[H_A(\omega)]\left[\frac{\partial[K_A]}{\partial\varphi}-\omega^2\frac{\partial[M_A]}{\partial\varphi}\right][H_A(\omega)] \tag{17}$$

where $[Z_A(\omega)]=[K_A]-\omega^2[M_A]$ is the dynamic stiffness matrix.

In the case of simple finite elements such as uniform beams, the derivatives of the elemental mass and stiffness matrices, $[M^e]$ and $[K^e]$, can be obtained analytically. For more advanced elements, the derivatives must be computed numerically by giving the design parameter φ a small increment h.

$$\begin{aligned}\frac{\partial[K_A^e(\varphi_i)]}{\partial\varphi_i}&\approx\frac{[K_A^e(\varphi_i+h)]-[K_A^e(\varphi_i)]}{h}\\ \frac{\partial[M_A^e(\varphi_i)]}{\partial\varphi_i}&\approx\frac{[M_A^e(\varphi_i+h)]-[M_A^e(\varphi_i)]}{h}\end{aligned} \tag{18}$$

The derivatives of the global mass and stiffness matrices, $[M_A]$ and $[K_A]$, can then be computed from those given in (18).

Method 2.

If (9) needs to be evaluated at a small number of frequencies, Method 1 will be more efficient in terms of computing time. On the other hand, if a large number of frequency points are used, it is probably better to compute two eigensolutions for each design parameter, one for the nominal value φ, and the other at value φ+ h. The theoretical response function matrices $[H_A(\varphi)]$ and $[H_A(\varphi+h)]$ can then be computed via modal summation. As before, the partial derivative with respect to φ is then given by:

$$\frac{\partial[H_A(\varphi_i)]}{\partial\varphi_i}\approx\frac{[H_A(\varphi_i+h)]-[H_A(\varphi_i)]}{h} \tag{21}$$

Numerical experience suggests that *h* must be chosen between 0.1% and 1%.

Whichever method is used, the partial derivatives $\frac{\partial[H_A]}{\partial\varphi},\frac{\partial[M_A]}{\partial\varphi},\frac{\partial[K_A]}{\partial\varphi}$ must be re-computed at each iteration since the nominal value of the design parameter will change at each iteration.

5. UPDATING CASE STUDY

The example of **Section 3** will again be used here to assess the performance of the correlation criteria based updating algorithm. The *Level 3* model will be used as the initial FE model and the simulated experimental data will be obtained from the *Level 4* model. Because of the different discretization levels, there is no one-to-one correspondence between the two models and no known errors are introduced to the initial FE model. The objective is to correct a given FE model (here *Level 3* model) using a given set of vibration test data (here obtained from the *Level 4* model) by assuming sufficient initial closeness between the two models. The correlation between the two models has already been discussed in **Section 3**. The 18 DOFs that were used in the correlation computations will also be used in the updating computations.

Here the choice of the design parameters is not obvious because the discrepancies between the two models are not due to specific artificial errors that were deliberately placed in the initial FE model. The updating frequency range, 200-2000 Hz, includes the first 9 modes of the structure. After some deliberation, the oil pan was subdivided into 9 regions, the element thicknesses of which were defined as the design parameters. The use of 1 design parameter per vibration mode is a conservative "rule of thumb" which is guided by previous numerical experience. All available frequency data, here 601 spectral points per FRF, were used. The order of the sensitivity matrix is thus 2x601 by 9, and the expected rank is 9. The algorithm was applied in an iterative fashion by solving equation (9) several times by using the last computed values of the design parameters. Convergence was obtained after 13 iterations and the variation of the 9 design parameters with the iteration number is shown in **Fig. 6**. As can be seen from **Fig. 6**, there is significant variation during the initial iterations, though the parameters appear to have converged to stable values after the 11^{th} iteration. Also, the requested thickness changes are observed to be within 8% of the nominal values.

The initially-predicted, updated and target FRFs are plotted in **Fig. 7**. It is clearly seen that the updated model is a distinct improvement over the initial one and the same information is also conveyed in **Table 3** where a natural frequency comparison is given. The average relative error is reduced to 0.4% from 2.5%.

Table3

Updated and measured natural frequencies (Hz) of the oil pan

Mode	Target	Level 3	Rel error (%)	Updated Level 3	Rel error (%)
1	242.4	249.3	2.8	240.0	-1.0
2	286.6	290.5	1.4	288.7	0.8
3	401.6	417.0	3.9	403.2	0.4
4	454.5	466.6	2.6	455.5	0.2
5	569.4	581.9	2.2	570.6	0.2
6	631.2	645.1	2.2	629.1	-0.3
7	725.3	744.1	2.6	725.4	0.0
8	759.0	779.6	2.7	759.9	0.1
9	803.8	822.7	2.3	806.1	0.3
Aver error	-	-	2.5	-	0.4

The values of the local amplitude criterion (LAC) were computed before and after updating and plotted in **Fig. 8**. Similarly the global shape and amplitude criteria, GSC and GAC, are plotted in **Fig. 9**. As expected, the quantification of the model improvement by all three criteria is satisfactory. However, it can be observed that the global correlation functions, especially GAC, exhibit a number of sudden drops around resonances. It is believed that the inclusion of damping in the FE model and its selection as a design parameter is likely to yield better results because it will then be possible to control and match the resonant amplitudes.

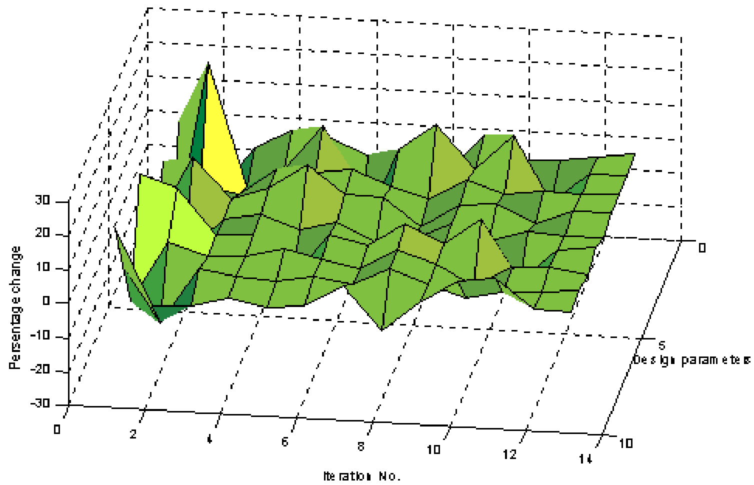


Fig. 6 Variation of design parameters with iteration number

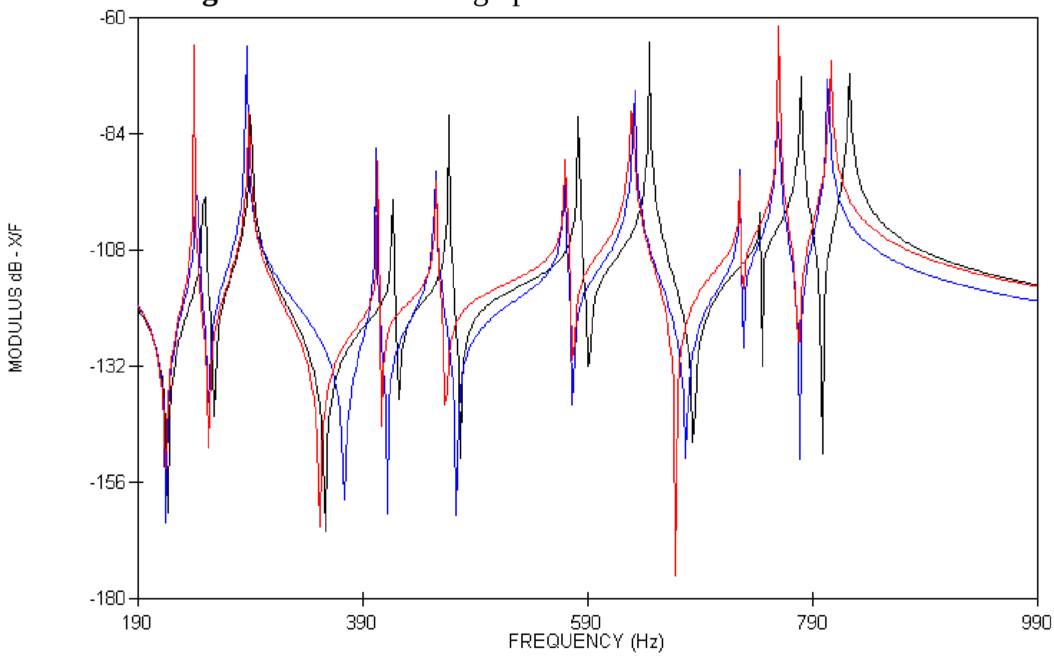
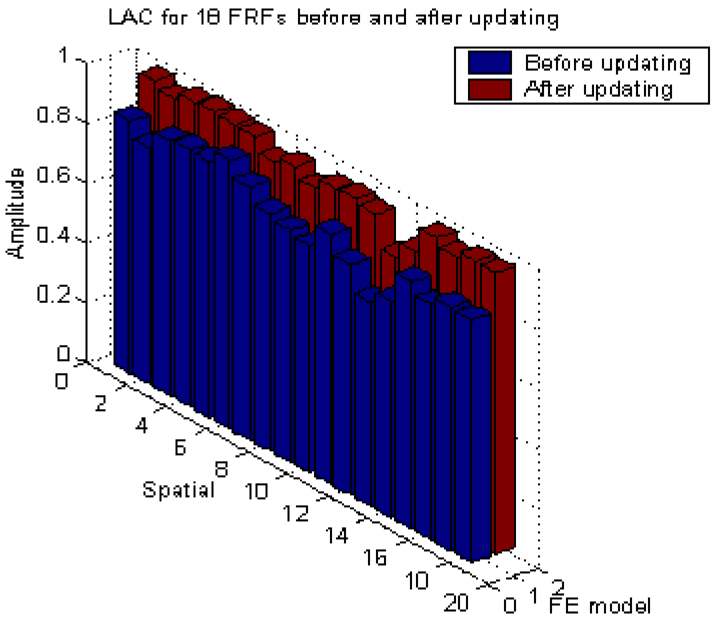


Fig. 7 Initially-predicted (black), updated (red) and target (blue) FRFs

Fig. 8 Local amplitude criterion (LAC) for 18 FRFs before and after updating



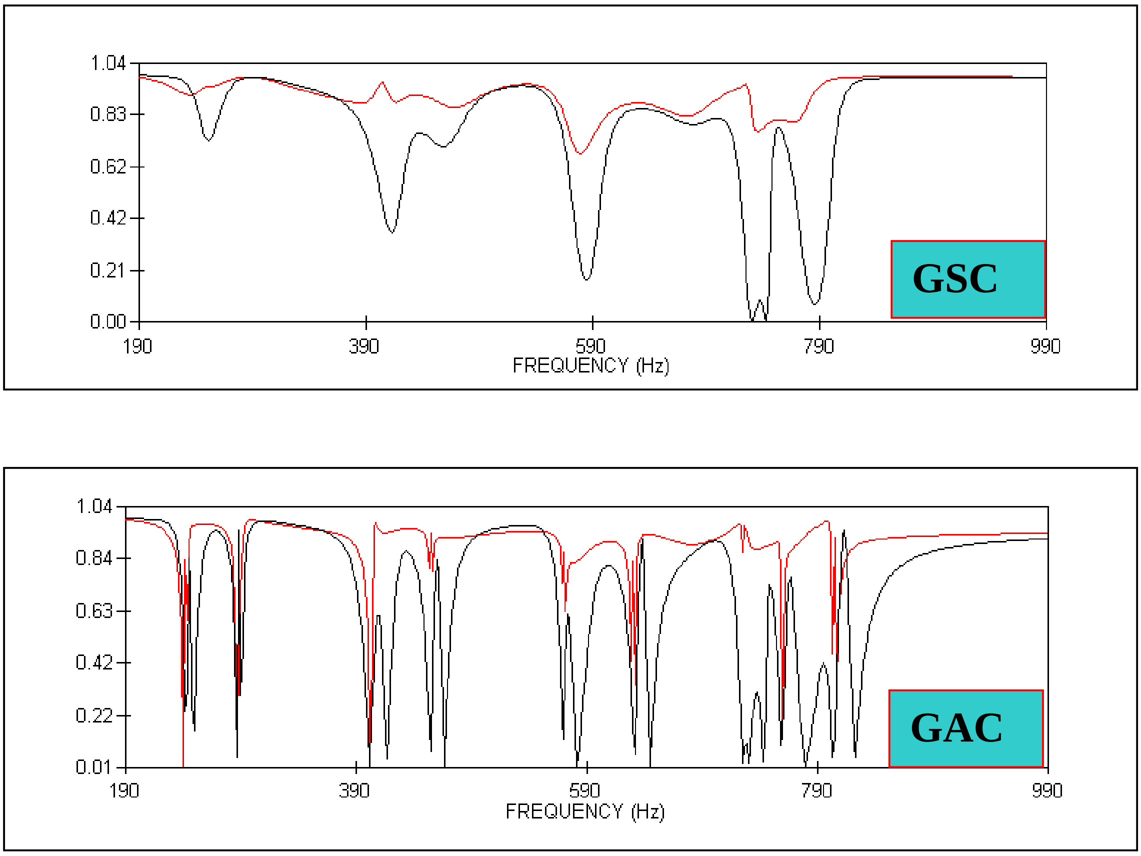


Fig. 9 Global shape and amplitude criteria, GSC and GAC, before and after updating

6. CONCLUDING REMARKS

(i) Based on shape and amplitude matching, local and global frequency-domain model correlation functions have been defined and used to quantify the initial model closeness. The results show that all four criteria, FAAC, LSC, GSC and GAC, are reliable numerical indicators of model correlation as a function of frequency, a feature which should help the analyst to make decisions about the usability/ updatability of the model for a specific frequency range.

(ii) The shape-based criteria are sensitive to mode shape differences but not to relative scales while the amplitude-based criteria attempt to match actual response amplitudes and hence they are sensitive to damping.

(iii) An updating algorithm has been devised using the sensitivity of the global correlation functions with respect to the selected design parameters. Since the functions can be defined for any number of FRFs, the size incompatibility between the theoretical and experimental models is no longer an obstacle. The shape and amplitude based functions interact with each other during the updating procedure, a feature that is somewhat analogous to predictor-corrector methods. Such a scheme has been found to be numerically robust.

(iv) Further studies, not reported here, indicate that the updating algorithm is relatively insensitive to noise. This is probably due to the fact that the updating equations can be made significantly over-determined by including all frequency-point information.

(v) Although the updating procedure is based on physically-realisable design parameters such as Young's modulus, density, element thickness, cross-section area, etc., updating can only be done in a global sense by tuning these parameters since, in the general case, there is no one-to-one correspondence between the theoretical and experimental models. In other words, actual modelling deficiencies are compensated by adjusting the selected design parameters, rather than identifying and eliminating such errors. Indeed, the use of physical design parameters versus non-dimensional multiplicative factors for FE model updating has been the subject of many debates. The results here seem to indicate that there is probably not much difference between the two approaches since successful updating was achieved by an artificial thickness adjustment, somewhat analogous to using a multiplicative factor.

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